Production of pions and clusters in heavy-ion collisions by the AMD+JAM approach

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Pion and Symmetry energy

➢ Motivation: We like to understand how Δ resonances and pions are affected by the dynamics of neutrons and protons in HIC.

Symmetry energy
soft / stiff

⟷

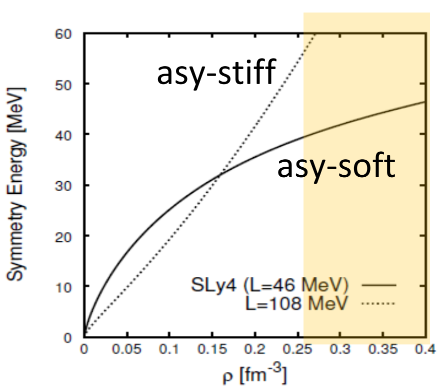
Nucleon
N/Z

⟷

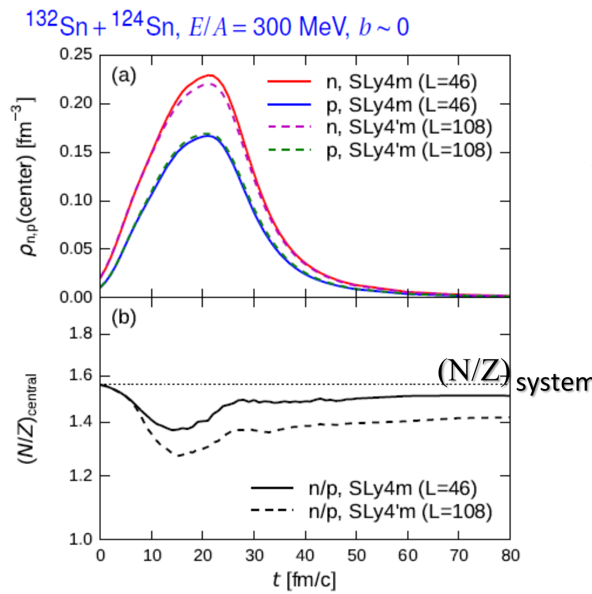
Δ resonance, Pion
Δ^-/Δ^{++}, π^-/π^+

Nucleon dynamics

$NN \leftrightarrow N\Delta$
$\Delta \leftrightarrow N\pi$

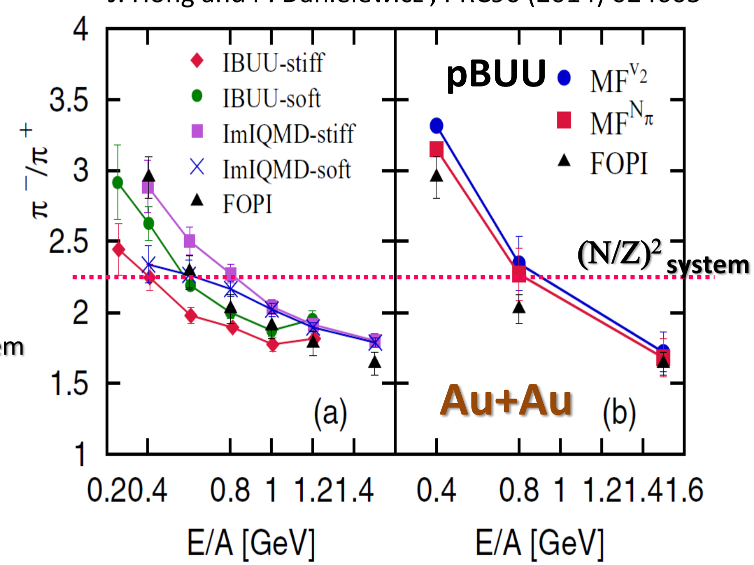


Interest:
High density $\rho \sim 2\rho_0$



Clear difference of N/Z in high density
due to different **S(ρ)**

J. Hong and P. Danielewicz , PRC90 (2014) 024605



Model predictions **do not agree**
Relation $\pi^-/\pi^+ \simeq (N/Z)^2$ **does not hold**

Our study

Pion production in $^{132}Sn + ^{124}Sn$ Collision @E/A=300 MeV

Some effects

- ✓ Symmetry energy (soft/stiff)
- ✓ Cluster correlation
- ✓ Pauli blocking (NEW)

Symmetry energy soft / stiff ↔ **Nucleon** N/Z ↔ **Δ resonance, Pion** Δ^{-}/Δ^{++}, π^{-}/π^{+}

$^{132}Sn + ^{124}Sn$, $^{108}Sn + ^{112}Sn$ Collision @270 MeV

- Experiment at RIKEN/RIBF
  SπRIT project

- ✓ Energy dependence
- ✓ Impact parameter dependence

➢ Theoretical Model:

AMD + JAM

AMD
- Nucleon dynamics
- Treatment of cluster correlation

JAM
- π, Δ production in the reaction process
- hadronic cascade model

Transport model (AMD + JAM)

- Coupled equations for $f_\alpha(\mathbf{r}, \mathbf{p}, t)$ ($\alpha = \mathrm{N}, \Delta, \pi$)

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N}{\partial \boldsymbol{p}} = I_N[f_N, f_{\Delta,\pi}]$$

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N, f_{\Delta,\pi}]$$

$I_{\mathrm{N}}[f_{\mathrm{N}}, f_{\Delta,\pi}]$:collision term

$\mathrm{N\,N} \to \mathrm{N\,N}$
$\mathrm{N\,N} \to \mathrm{N}\,\Delta$
$\mathrm{N}\,\Delta \to \mathrm{N\,N}$
$\Delta \to \mathrm{N}\,\pi$
$\mathrm{N}\,\pi \to \Delta$ ... etc.

Our model: JAM coupled with AMD

Perturbative treatment of pion and Δ particle production

$$I_N = I_N^{\mathrm{el}}[f_N, 0] + \lambda I'_N[f_N, f_{\Delta,\pi}]$$

$f_{\Delta,\pi} = O(\lambda)$: Δ and pion productions are rare

$f_N = f_N^{(0)} + \lambda f_N^{(1)} + \ldots$

- **Nucleon f_{N} : Zeroth order equation**

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N^{(0)}, 0]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{p}} = I_N^{\mathrm{el}}[f_N^{(0)}, 0]$$

Solved by AMD

- **Δ particle f_Δ and pion f_π : First order equation**

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]$$

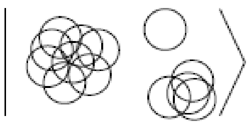
Solved by JAM for given $f_{\mathrm{N}}^{(0)}$

Transport model (AMD + JAM)

➢ AMD (Antisymmetrized Molecular Dynamics)

A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, PTP87 (1992) 1185

- AMD wave function



$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

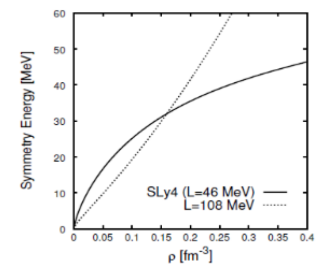
$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \mathrm{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Solve the time evolution of the wave packet centroids Z

✓ Effective interaction

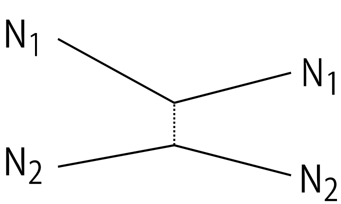


Skyrme force

- Turn on/off Cluster correlation

- Without Cluster

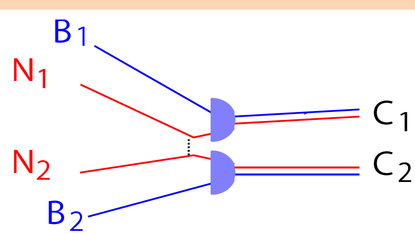
N1 + N2 -> N1 + N2



N1, N2 : Colliding nucleons

- With Cluster

N1 + B1 + N2 + B2 -> C1 + C2



N1, N2: Colliding nucleons

B1, B2: Spectator nucleons/clusters

C1, C2: N, (2N), (3N), (4N) (up to α cluster)

Transport model (AMD + JAM)

- Nucleon test Particles

Test particles ($\mathbf{r}_1$, $\mathbf{p}_1$), ($\mathbf{r}_2$, $\mathbf{p}_2$), …, ($\mathbf{r}_A$, $\mathbf{p}_A$) are generated following the Wigner function $f^{\tau}_{\mathrm{AMD}}(\mathbf{r}, \mathbf{p})$ for τ = neutron or proton

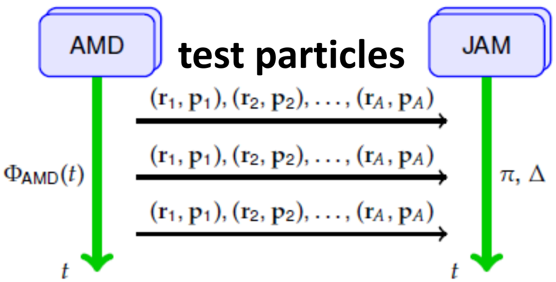
$$f^{\tau}_{\mathrm{AMD}}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2} \times 2^3 \sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

$$\boldsymbol{R}_{jk} = (\boldsymbol{Z}^*_j + \boldsymbol{Z}_k)/\sqrt{\nu}$$

$$\boldsymbol{P}_{jk} = 2i\hbar\sqrt{\nu}(\boldsymbol{Z}^*_j - \boldsymbol{Z}_k)$$

$$B_{jk} = \langle\varphi_j|\varphi_k\rangle$$

We send **nucleon test particles ($\mathbf{r}_1$, $\mathbf{p}_1$), ($\mathbf{r}_2$, $\mathbf{p}_2$), …, ($\mathbf{r}_A$, $\mathbf{p}_A$)** from AMD to JAM at every 2 fm/c with corrections for the conservation of baryon number and charge.



➢ JAM (Jet AA Microscopic transport model)

Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, PRC61 (2000) 024901

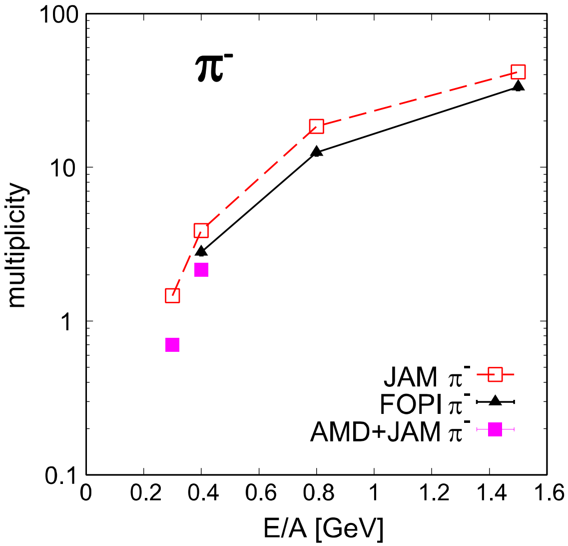
- Applied to high-energy collisions (1 ～ 158 A GeV)
- Hadron-Hadron reactions are based on experimental data and the detailed balance.
- No mean field (default)
- *s*-wave pion production (NN→NNπ) is turned off. … etc.

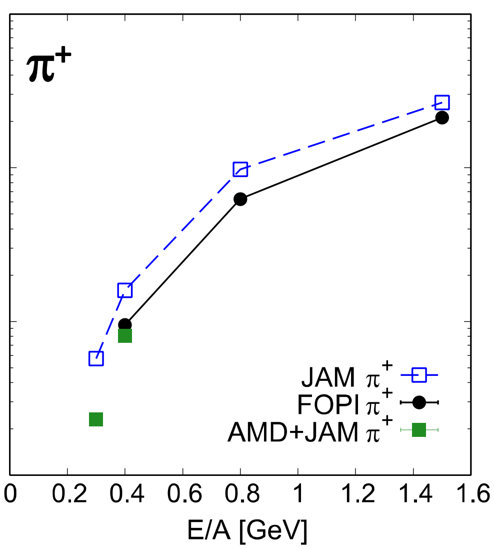
Pion Calcutions in central Au+Au collisions

- Pion multiplicity

by transport model (AMD + JAM)

with cluster (asy-soft)

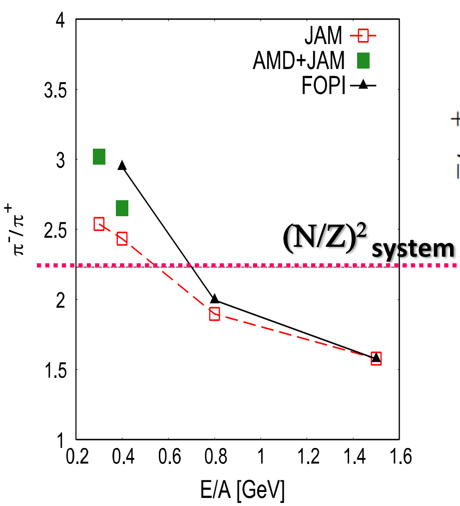


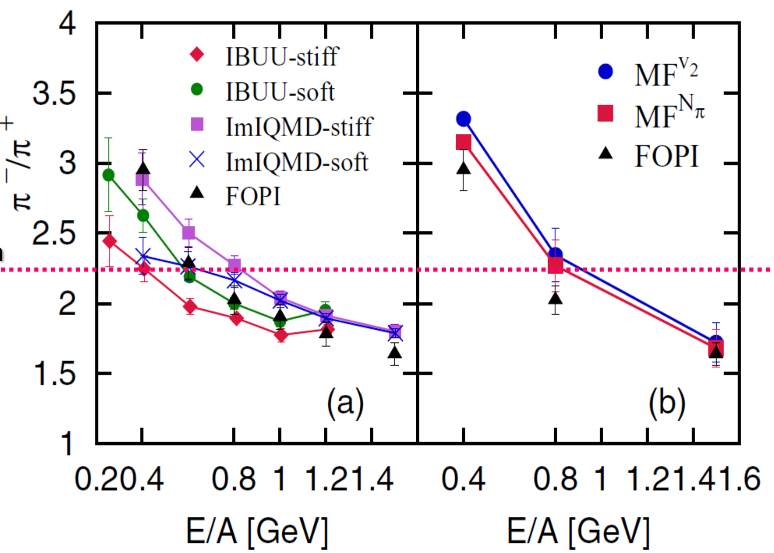


✓ Our calculation almost reproduces the experimental data reasonably well

✓ Pion ratios are also larger than $(N/Z)^2_{system}$

- Pion ratio





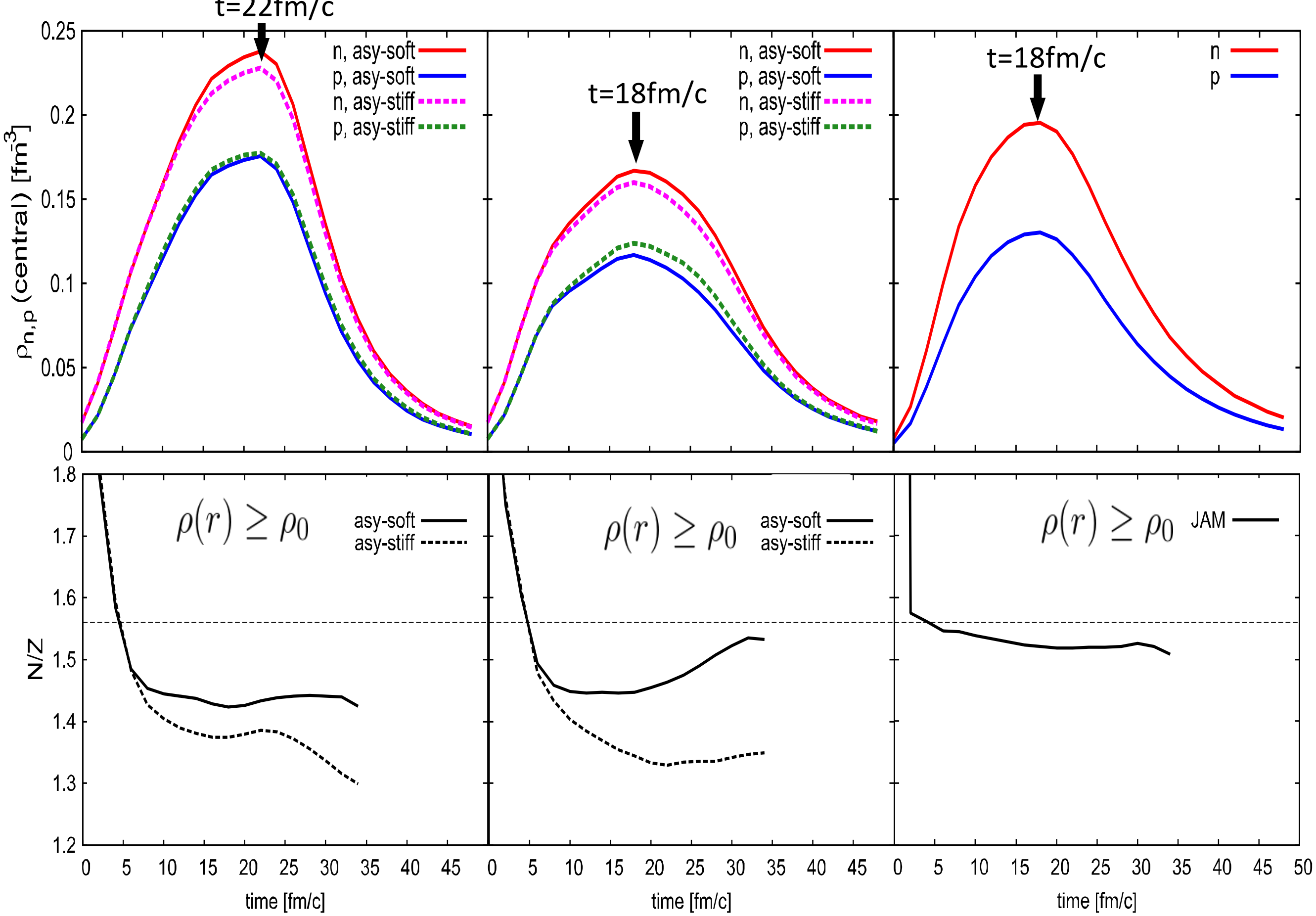
J. Hong and P. Danielewicz , PRC90 (2014) 024605

Exp. Data: Reisdorf *et al.*, NPA 848 (2010) 366

Dynamics of neutrons and protons

$^{132}Sn + ^{124}Sn$ Collision @E/A=300 MeV

- with cluster
- without cluster
- JAM



- ✓ Density maximum is different for cases with or without cluster
- ✓ Clear difference of N/Z ratio due to different symmetry energy
- ✓ Especially symmetry energy effect is weaker if there is cluster correlation

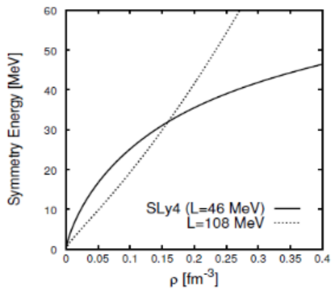
Calculation set:

AMD + JAM

1. with cluster (asy-soft)
2. with cluster (asy-stiff)
3. without cluster (asy-soft)
4. without cluster (asy-stiff)

5. JAM (no mean field)

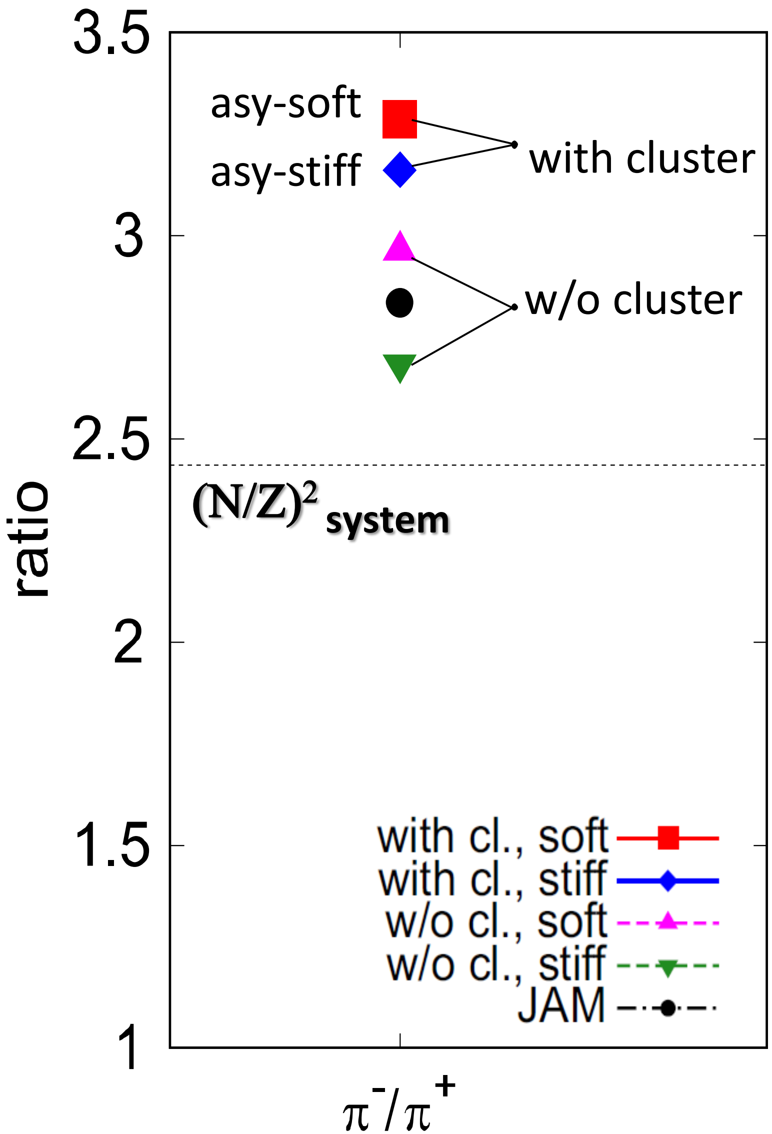
asy-soft : L=46 (SLy4)
asy-stiff : L=108



Effective interaction:
Skyrme force

Final π^-/π^+ ratio

^{132}Sn + ^{124}Sn Collision @E/A=300 MeV



1. Symmetry energy dependence S(ρ)
   π^-/π^+ ratio with soft S(ρ) is larger
   –> Similar result to IBUU

2. Model dependence of nucleon dynamics
   S(ρ) effect is weaker with cluster correlations

3. π^-/π^+ ratio > $(N/Z)^2_{\text{system}}$

⇒What is the origin of these behaviors?

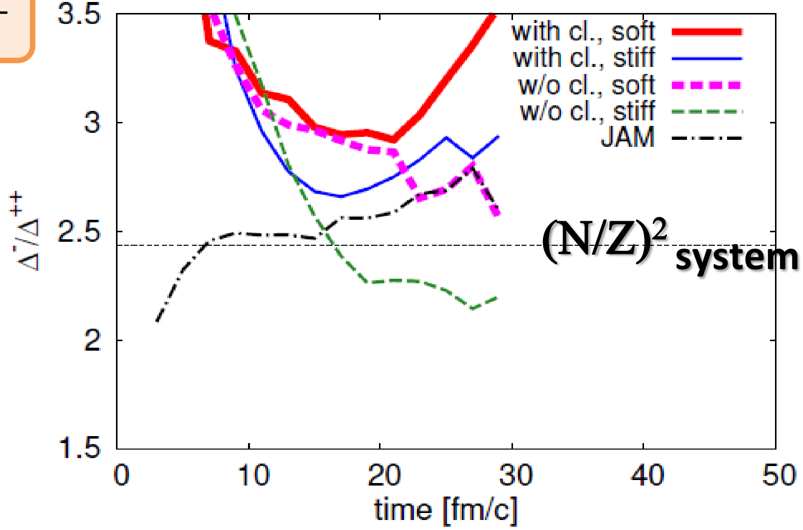
$NN \leftrightarrow N\Delta \qquad \Delta \leftrightarrow N\pi$

We study what kind of information of nucleon is carried by Δ resonances.

Relation between N/Z and Δ^-/Δ^{++}

Nucleon N/Z ↔ ? ↔ Δ resonance Δ^-/Δ^{++}

Δ^-/Δ^{++}



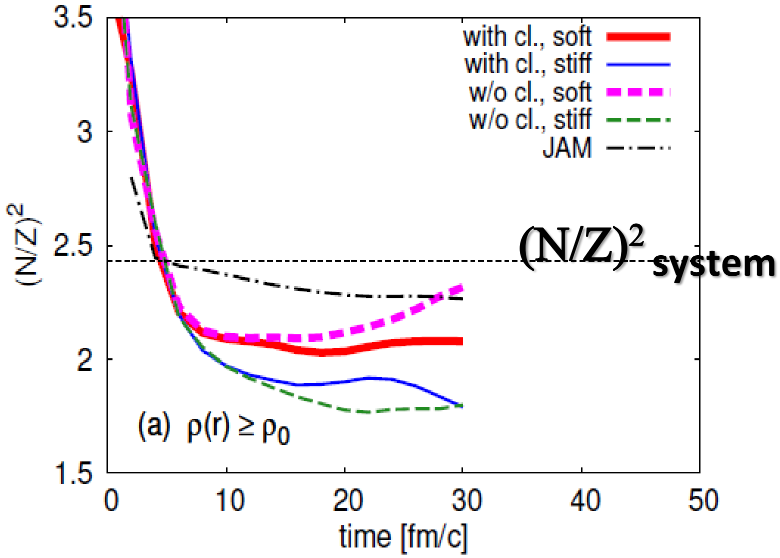
Simple expectation: $\Delta^-/\Delta^{++} \sim (N/Z)^2$

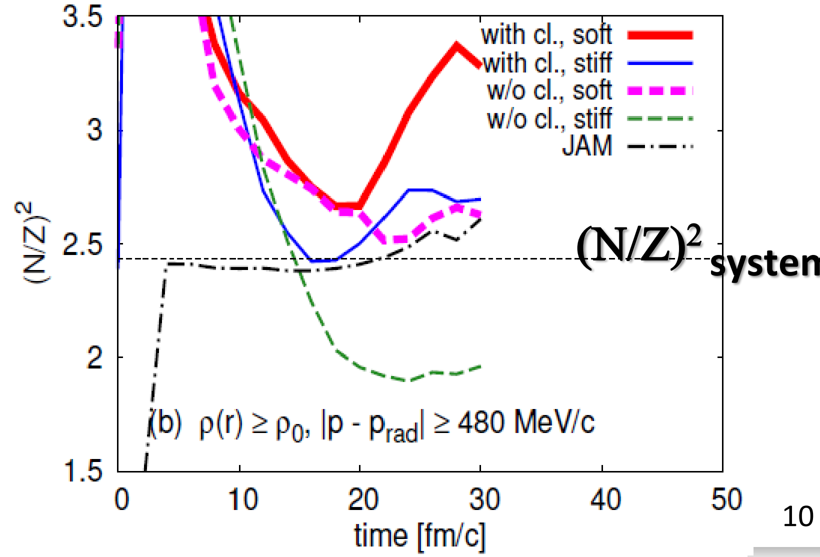
$$\frac{\Delta^-}{\Delta^{++}} = \frac{\text{Rate}(nn \to n\Delta^-)}{\text{Rate}(pp \to p\Delta^{++})}$$

(The collective radial momentum $\boldsymbol{p}_{rad}$ is subtracted)

$(N/Z)^2_\rho$ Nucleons in the sphere $\rho(r) \geqq \rho_0$ centered at CM.

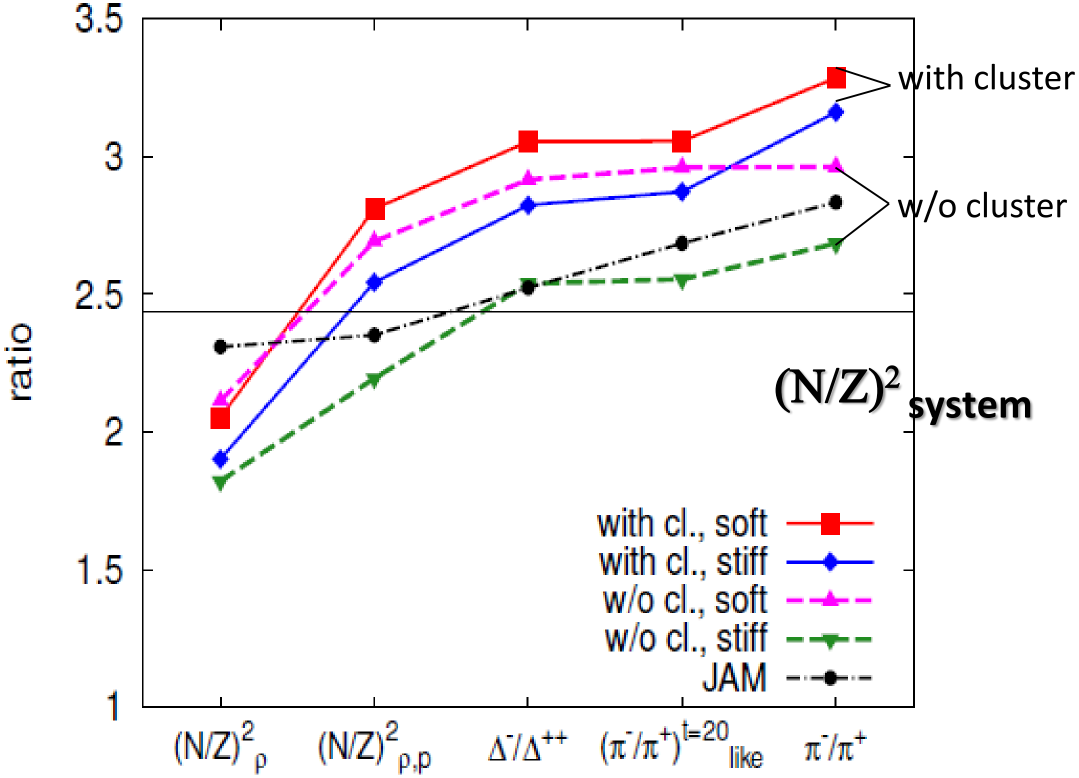
$(N/Z)^2_{\rho,\,p}$ Nucleons in the sphere $\rho(r) > \rho_0$ with **high momentum**





Final π^-/π^+ ratio

- From nucleons to pion ratios



Representative ratios:

$$\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$$

$$\frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \to p\Delta^-)dt}{\int_0^\infty (pp \to n\Delta^{++})dt}$$

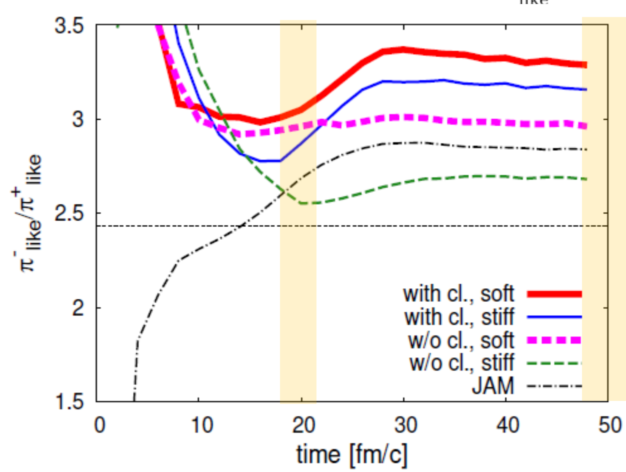
N(t), Z(t) : Numbers of nucleon which satisfy the conditions

- $\Delta^-/\Delta^{++} \sim (\pi^-/\pi^+)^{t=20}_{like}$
- Final stage:
  π^-/π^+ is modified from $(\pi^-/\pi^+)^{t=20}_{like}$
  - ✓ $S(\rho)$ effect: 30% weaker
  - ✓ Cluster correlation → π^-/π^+ up

=> Cluster correlation played important roles for the pions.

$$\pi^-_{like} = \pi^- + \Delta^- + \frac{1}{3}\Delta^0$$

$$\pi^+_{like} = \pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+$$

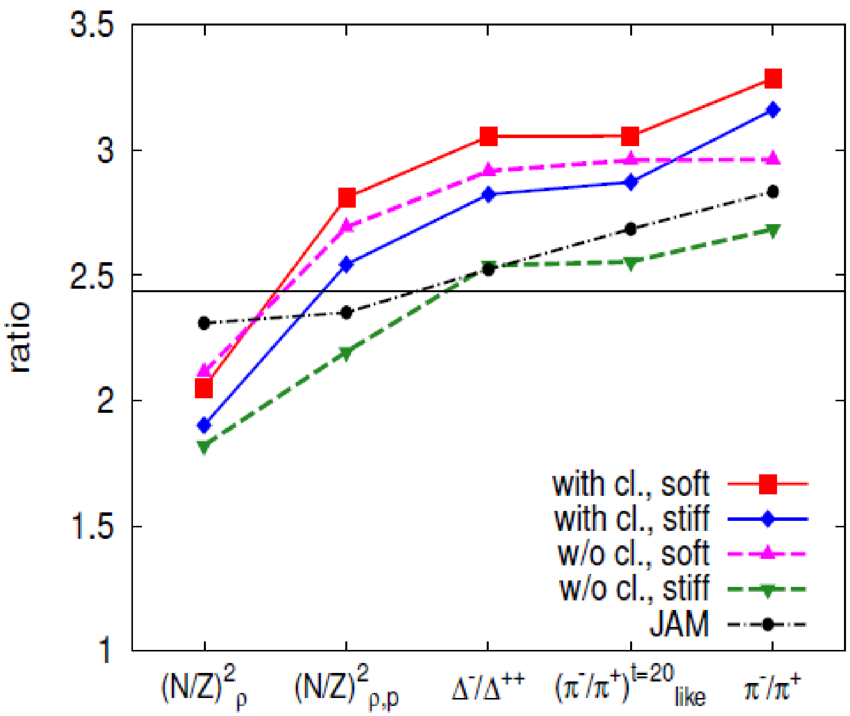


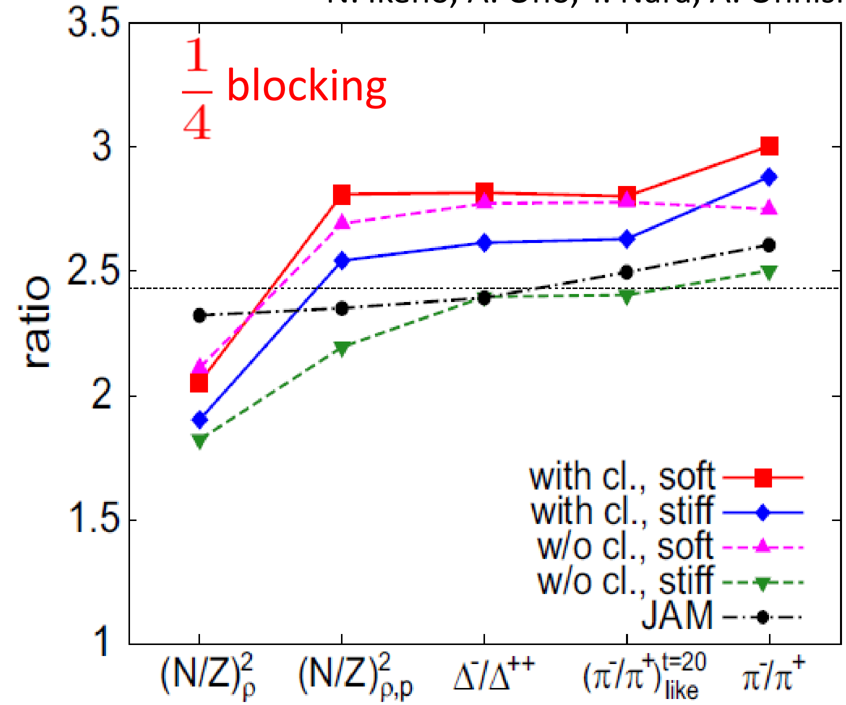
Pauli-blocking effect

Recently, we found that Pauli-blocking effect is important.

PRC93, 044612 (2016)

N. Ikeno, A. Ono, Y. Nara, A. Ohnishi





$$f^\tau(\boldsymbol{r},\boldsymbol{p}) = 4\sum_{j\in\tau} e^{-(\boldsymbol{r}-\boldsymbol{r}_j)^2/2L-2L(\boldsymbol{p}-\boldsymbol{p}_j)^2/\hbar^2}$$

(τ = neutron or proton)

$$\frac{1}{4}\times 4\sum_{j\in\tau} e^{-(\boldsymbol{r}-\boldsymbol{r}_j)^2/2L-2L(\boldsymbol{p}-\boldsymbol{p}_j)^2/\hbar^2}$$

π^-/π^+ and Δ^-/Δ^{++} ratios change due to Pauli-blocking effect.

Pauli-blocking effect

Pauli-blocking for the final nucleon(s)
in two-body collisions

<u>π^- production</u>

$$nn \rightarrow \underline{p}\Delta^-$$
$$\Delta^- \rightarrow \underline{n}\pi^-$$

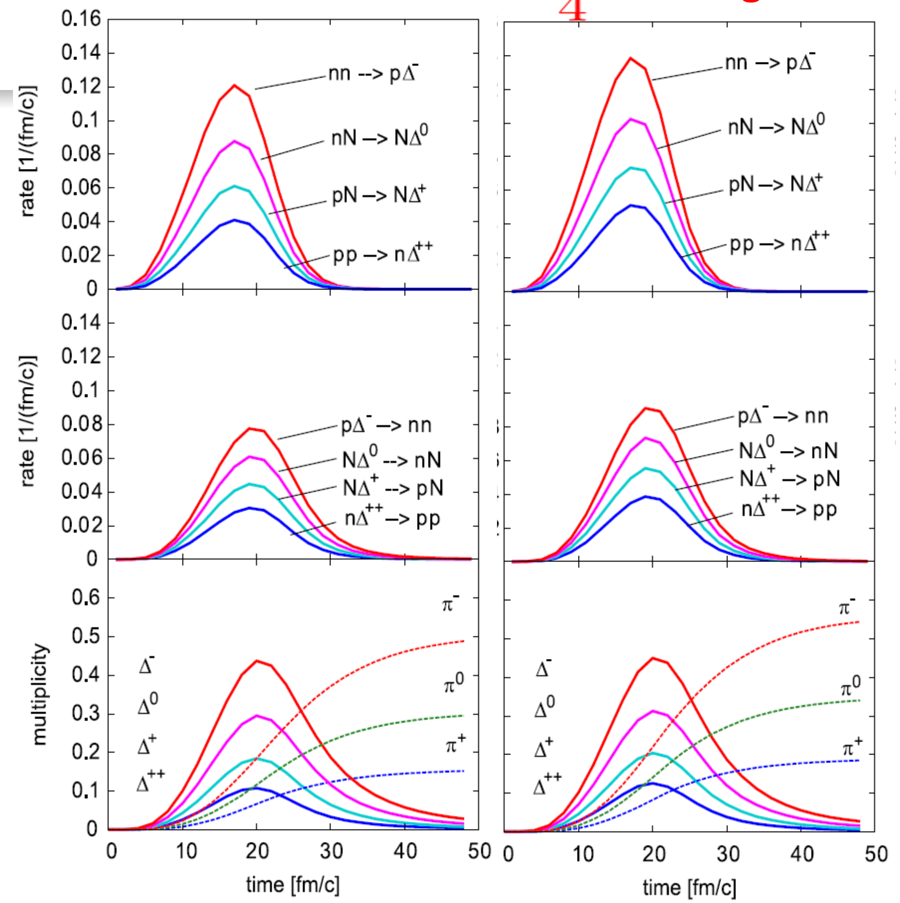
<u>π^+ production</u>

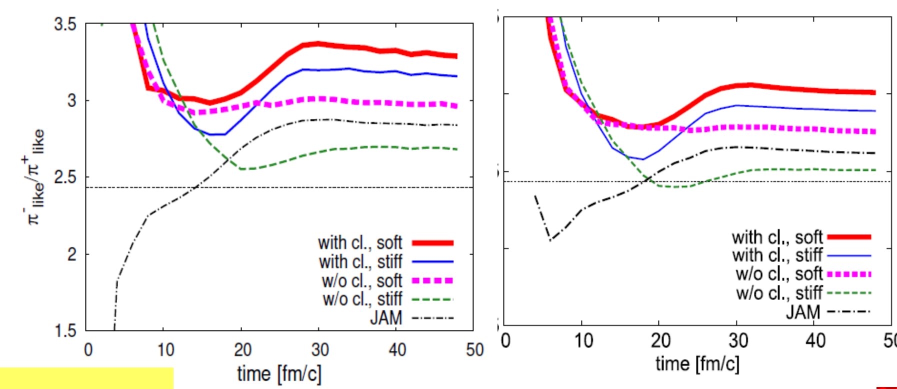
$$pp \rightarrow \underline{n}\Delta^{++}$$
$$\Delta^{++} \rightarrow \underline{p}\pi^+$$

When Effect of Pauli-blocking is stronger
-> nucleons are blocked stronger
-> Δ and π multiplicities are smaller

n-rich system
-> neutrons are blocked stronger
-> Δ^{++} and π^+ multiplicities are smaller
-> π^-/π^+ ratio is larger

$\frac{1}{4}$ blocking





Pauli-blocking effect played an important roles for the pions.

Methods for Pauli blocking factor

➢ Use *f* of AMD for Pauli blocking

The Wigner function calculated for the AMD wave function, for τ = neutron or proton, is

$$f^{\tau}_{\mathrm{AMD}}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2} \times 2^3 \sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

$$\boldsymbol{R}_{jk} = (\boldsymbol{Z}^*_j + \boldsymbol{Z}_k)/\sqrt{\nu}$$

$$\boldsymbol{P}_{jk} = 2i\hbar\sqrt{\nu}(\boldsymbol{Z}^*_j - \boldsymbol{Z}_k)$$

$$B_{jk} = \langle\varphi_j|\varphi_k\rangle$$

$P_{\mathrm{block}} = f^{\tau}_{\mathrm{AMD}}(\mathbf{r}_i, \mathbf{p}'_i)$ for the final phase-space point $(\mathbf{r}_i, \mathbf{p}'_i)$.

Test particles $\{(\mathbf{r}_i, \mathbf{p}'_i)$; , i=1,2, … ,A$\}$ are generated with the probability distribution $f^{\tau}_{\mathrm{AMD}}(\mathbf{r},\mathbf{p})$ and sent to JAM.

➢ Do Pauli blocking within JAM

$P_{\mathrm{block}} = f^{\tau}_{\mathrm{JAM}}(\mathbf{r}_i, \mathbf{p}'_i)$ with

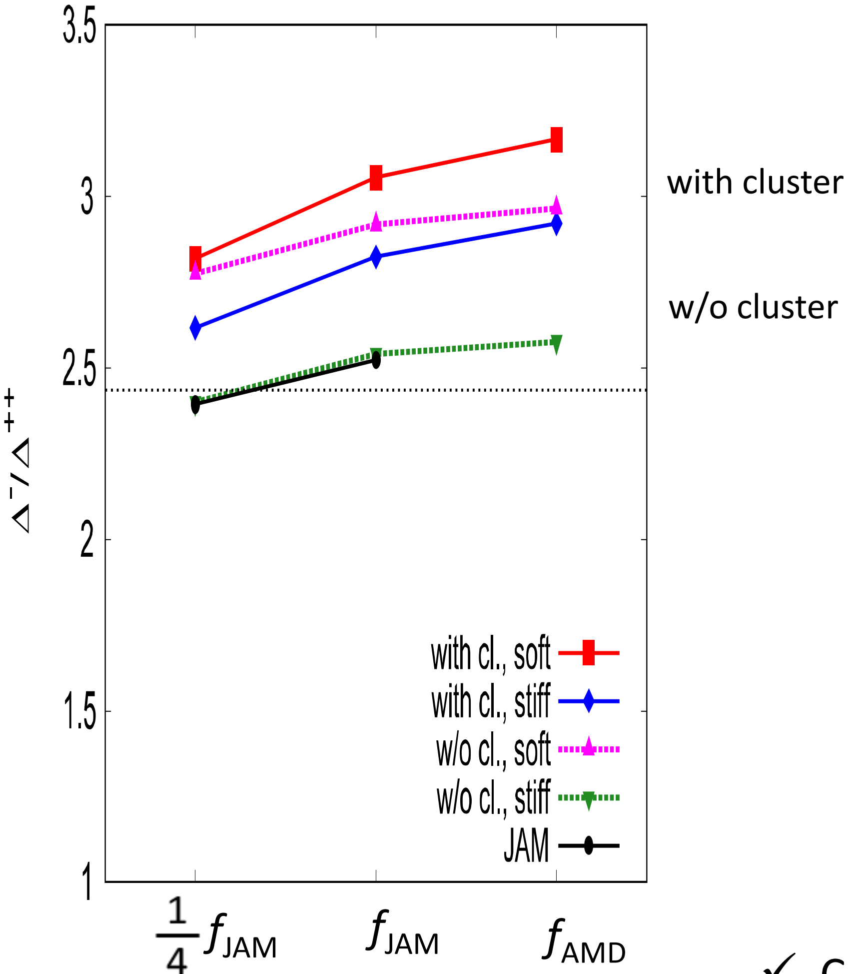
$$f^{\tau}_{\mathrm{JAM}}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2} \times 2^3 \sum_{j\in\tau} e^{-(\boldsymbol{r}-\boldsymbol{r}_j)^2/2L-2L(\boldsymbol{p}-\boldsymbol{p}_j)^2/\hbar^2}$$

L=2.0 fm^2

=> We compare $\frac{1}{4}f_{\mathrm{JAM}}$, f_{JAM} and f_{AMD}, to see the effect and importance of Pauli blocking treatment

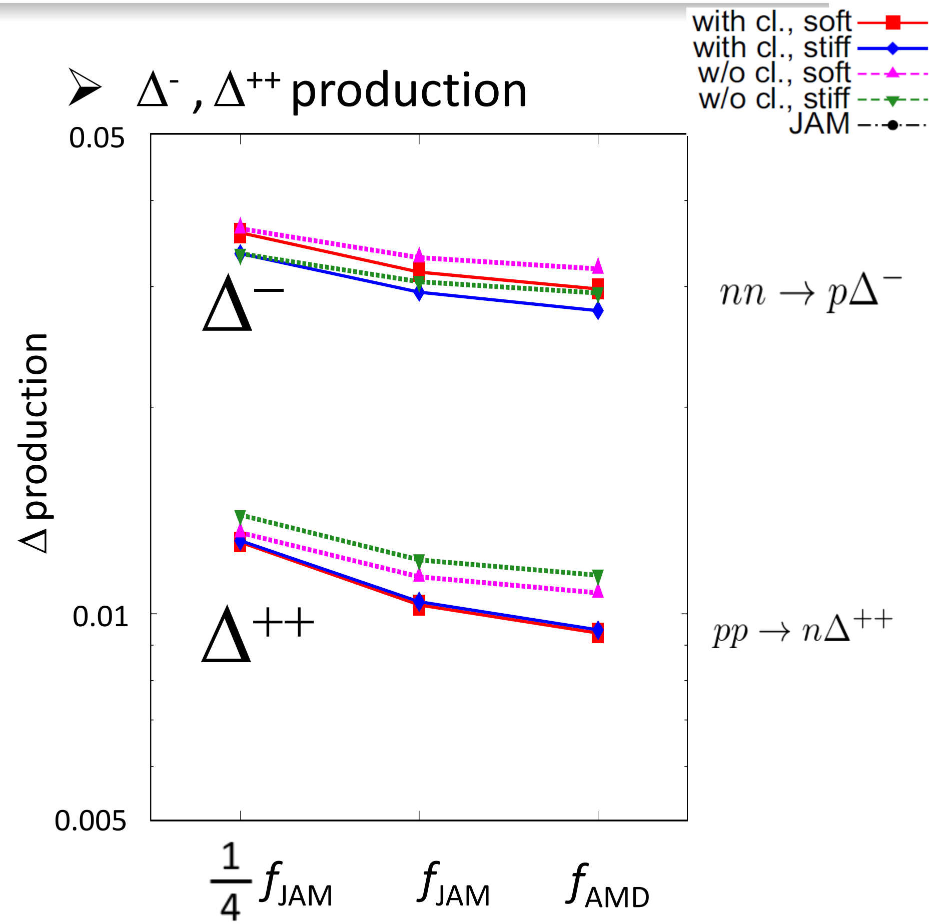
Different treatments for Pauli-blocking

- Production ratio Δ^-/Δ^{++}



$$\frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \to p\Delta^-)dt}{\int_0^\infty (pp \to n\Delta^{++})dt}$$

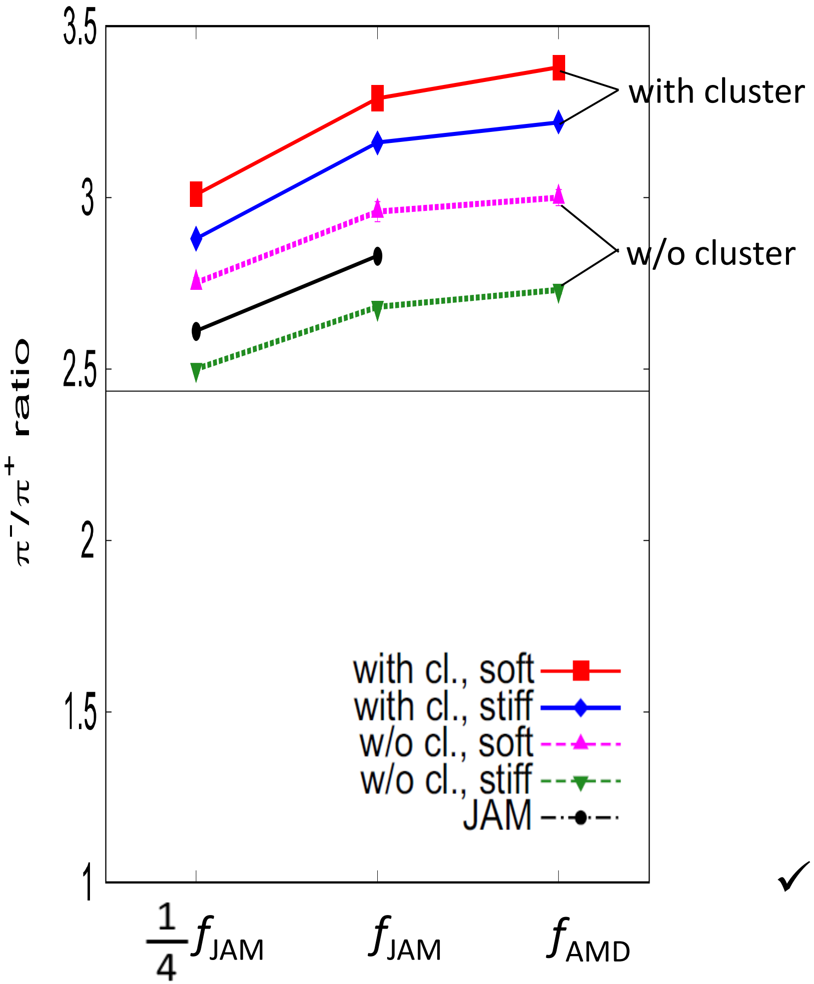
- Δ^-, Δ^{++} production



✓ Clear difference of ratio due to different treatments for Pauli-blocking

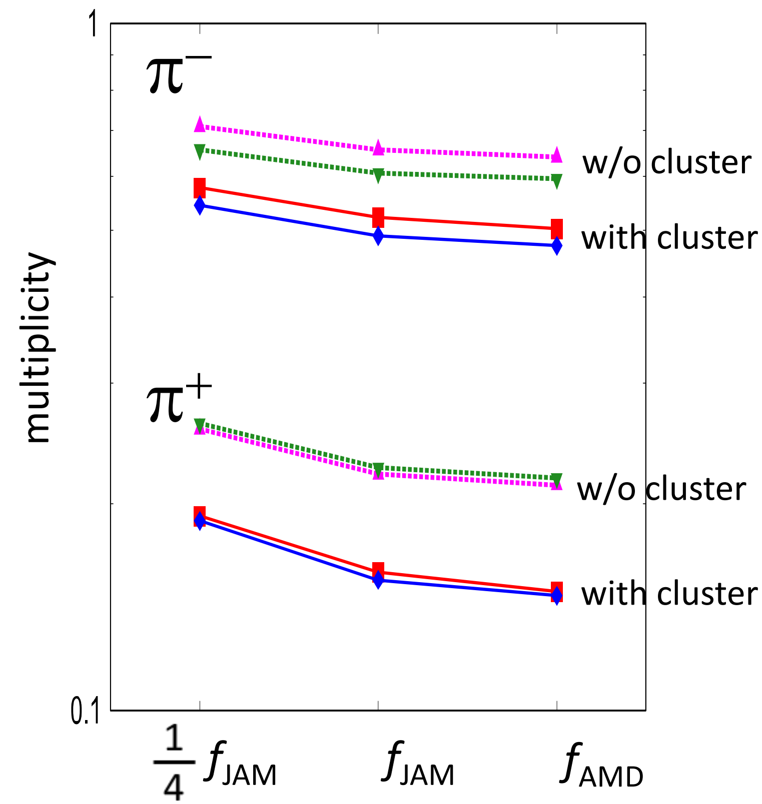
Different treatments for Pauli-blocking

- Final π^-/π^+ ratio



$NN \leftrightarrow N\Delta \quad \Delta \leftrightarrow N\pi$

- Multiplicities of π^-, π^+

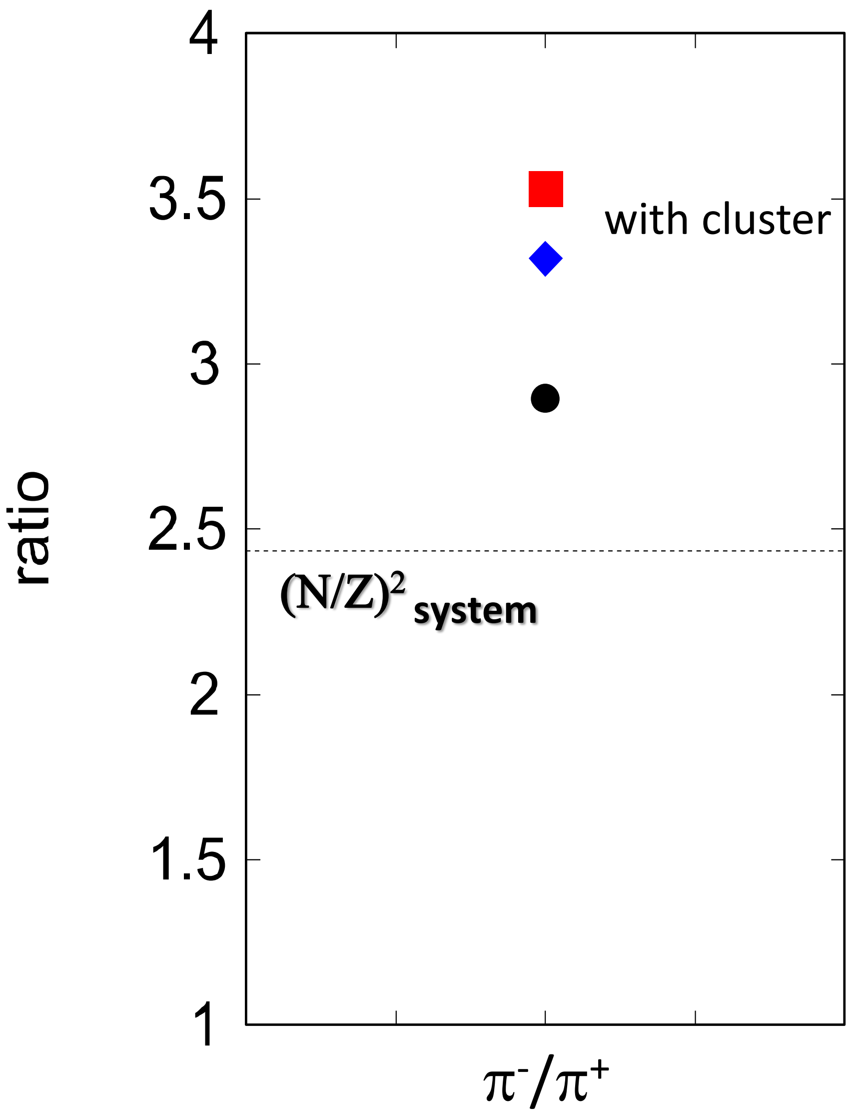


- ✓ Pauli blocking effect is stronger for π^+ in particular when cluster correlation is switched on.
- ✓ Symmetry energy effect S(ρ) is stronger for π^- than for π^+.
- ✓ Cluster correlation effect is stronger for π^+.

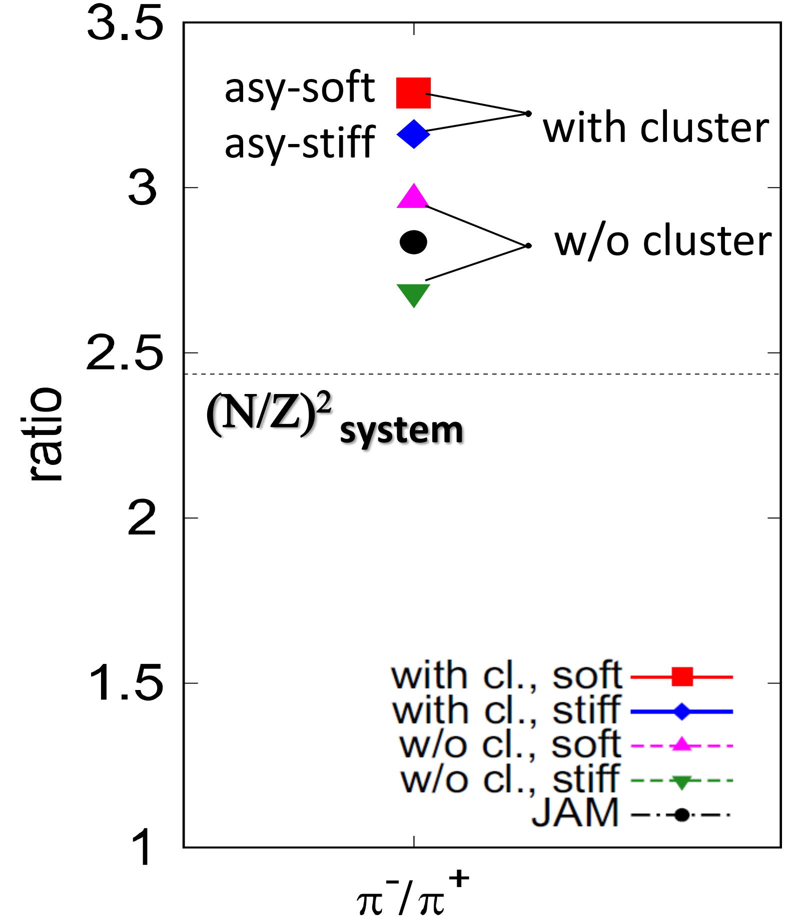
Energy dependence of $^{132}Sn + ^{124}Sn$

➢ Final π^-/π^+ ratio

E/A=270MeV : Experiment at SπRIT project

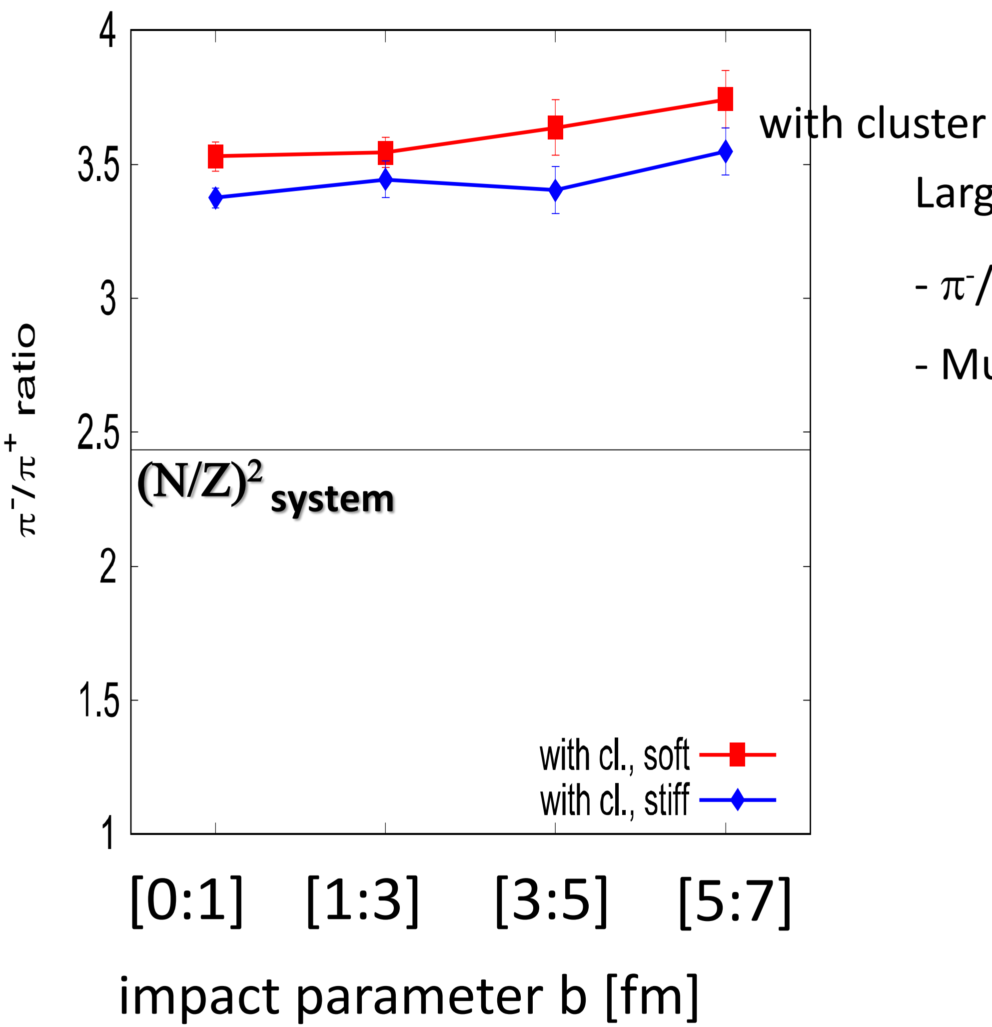


E/A=300MeV



Impact parameter dependence

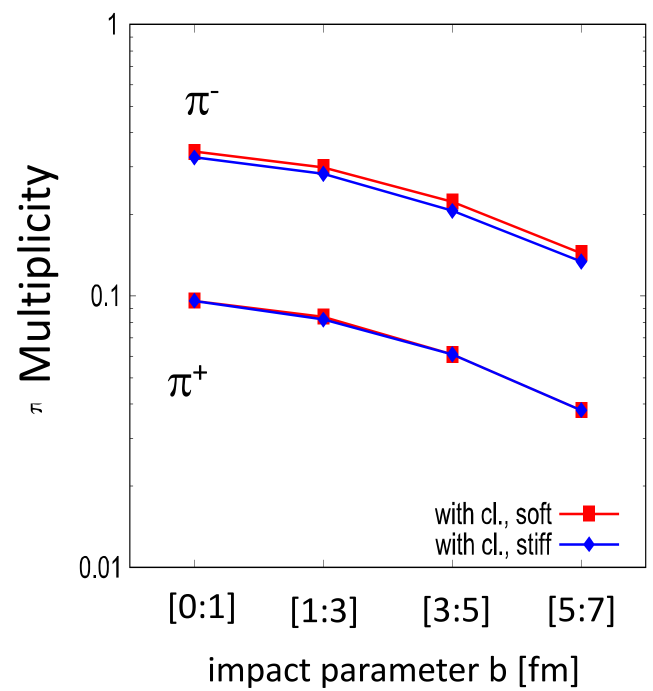
Final π^-/π^+ ratio



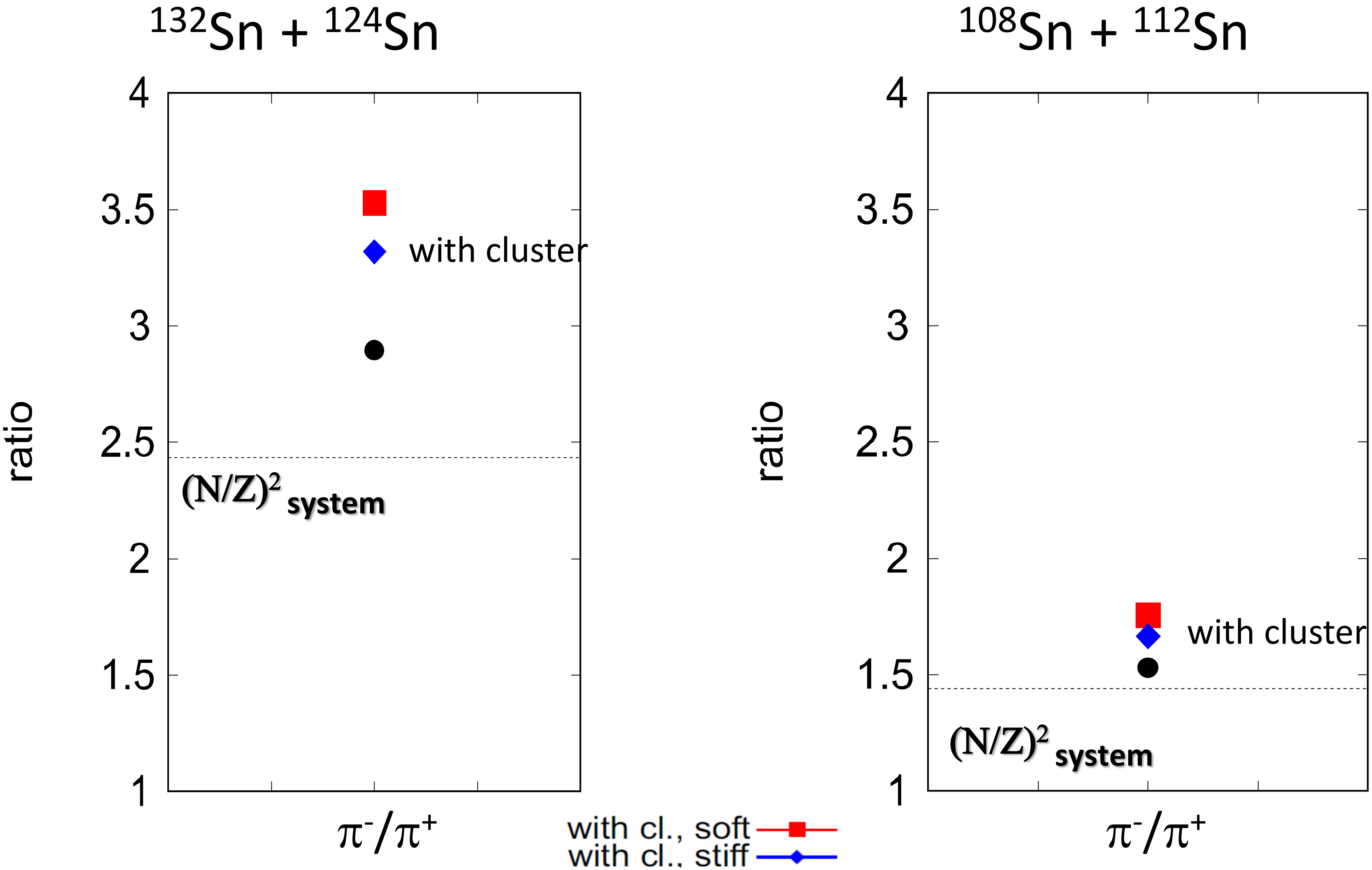
$^{132}Sn + ^{124}Sn$ @E/A=270MeV

Larger impact parameter b

- π^-/π^+ ratio is larger because of n-rich probably
- Multiplicities of π^-, π^+ are smaller



Different system @ E/A=270MeV : Experiment at SπRIT project



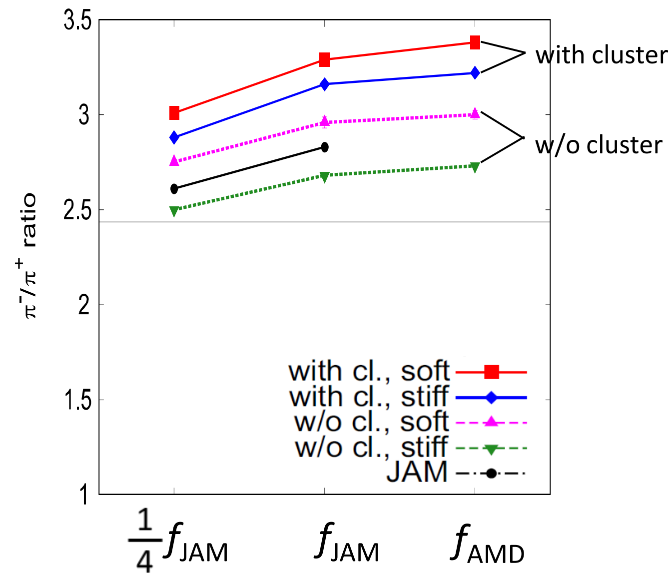
Summary: Pion production in Sn+Sn collisions by AMD+JAM

Motivation: To understand the mechanism how pions are produced reflecting the dynamics of neutrons and protons

- Pion ratio certainly carries the information on neutrons and protons
  - ✓ π^-/π^+ and Δ^-/Δ^{++} ratios are related to the $(N/Z)^2$ in high-ρ and high-p region
  - ✓ In the final stage, π^-/π^+ ratio is modified from $(\pi^-/\pi^+)^{t=20}_{\text{like}}$

Important effects for pions

- Symmetry energy (soft/stiff)
  - ✓ π^-/π^+ ratio with soft S(ρ) is larger
- Cluster correlation
  - ✓ S(ρ) effect is weaker with cluster correlations
- Pauli-blocking effect
  - ✓ Multiplicity and ratio change



Future work:

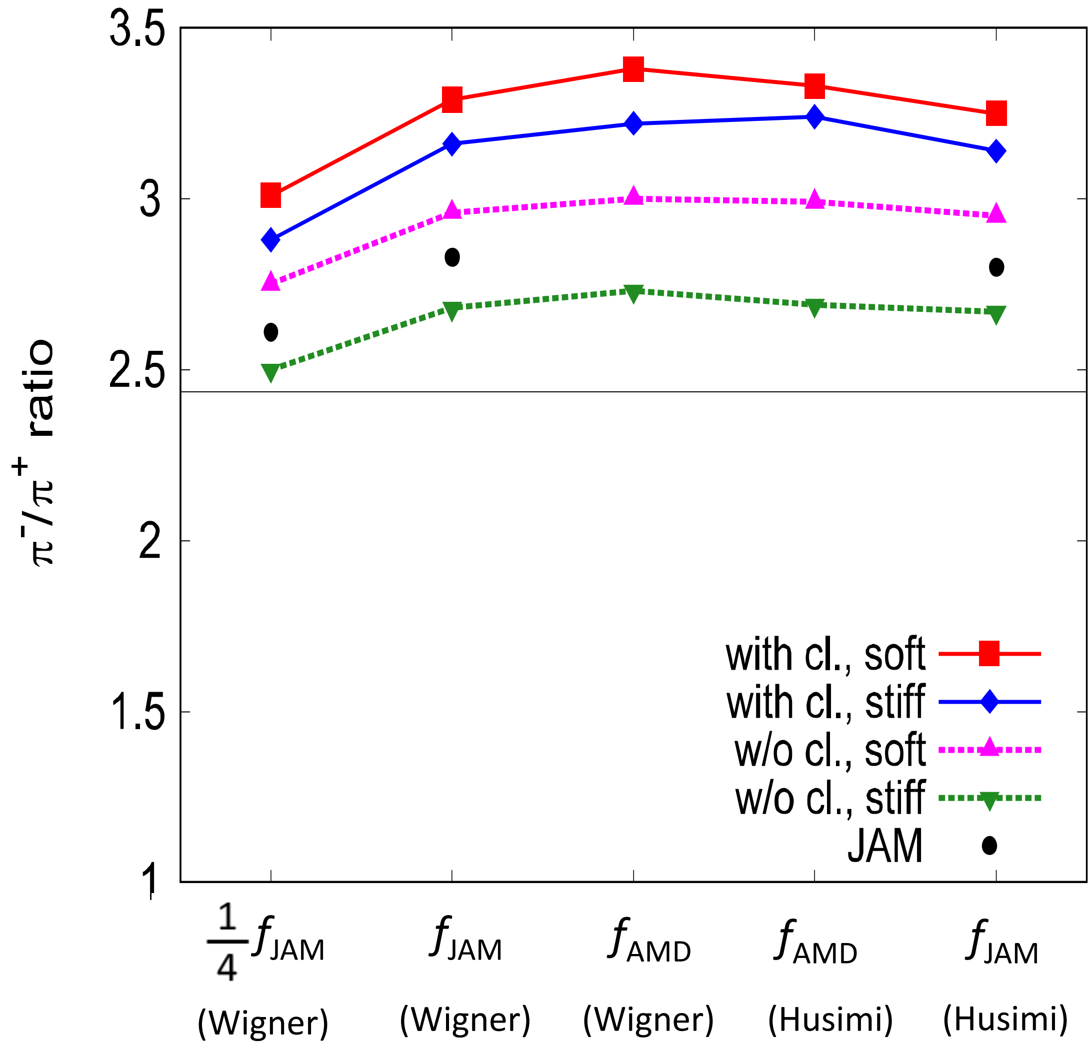
We calculate to compare with experimental data.

We need to investigate pions but also other observables (cluster correlation)

- Δ resonance production threshold

Different treatments for Pauli-blocking

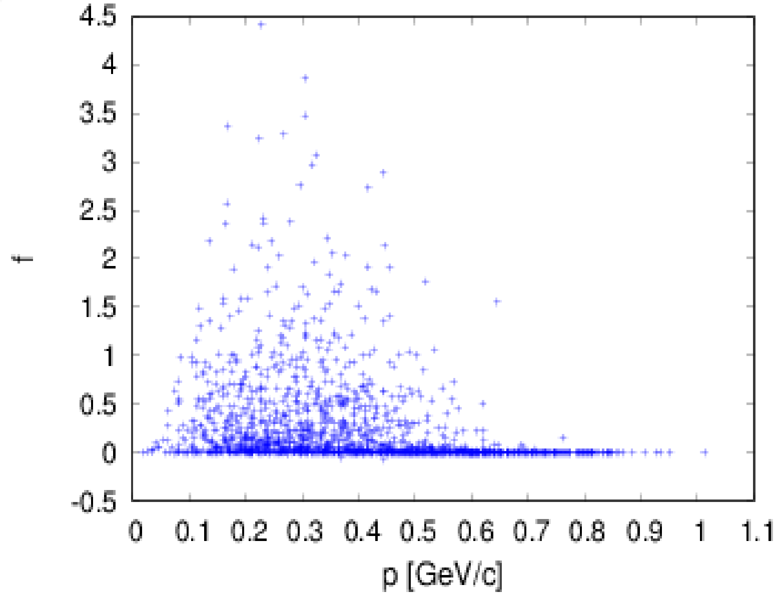
- Final π^-/π^+ ratio



Phase space distribution function f

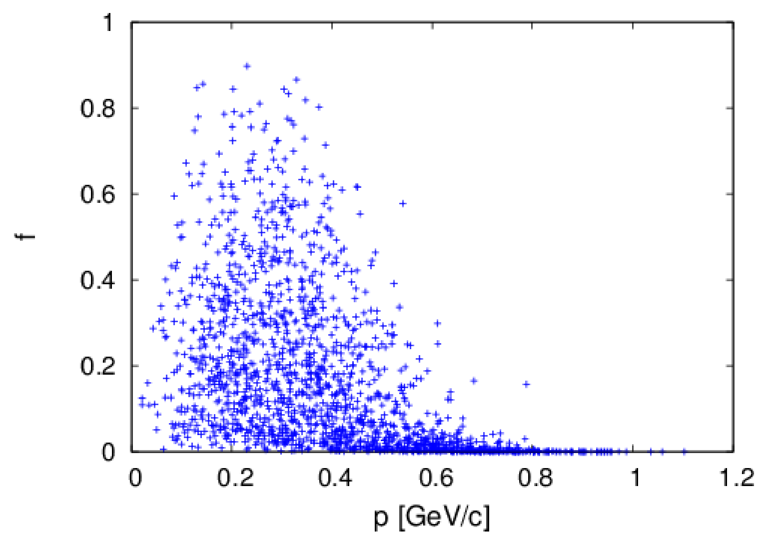
Wigner function

$$f^{\tau}_{\rm AMD}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2} \times 2^3 \sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$



Husimi function

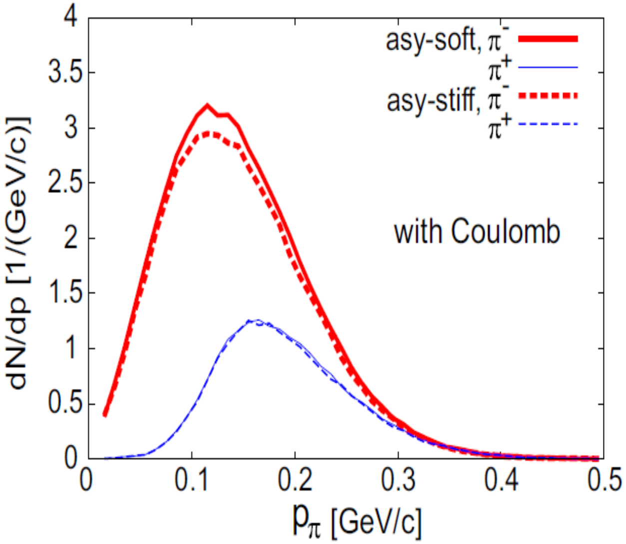
$$f^{\tau}_{\rm AMD}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2} \times \sum_{j\in\tau}\sum_{k\in\tau} e^{-\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/4\hbar^2\nu} B_{jk}B^{-1}_{kj}$$



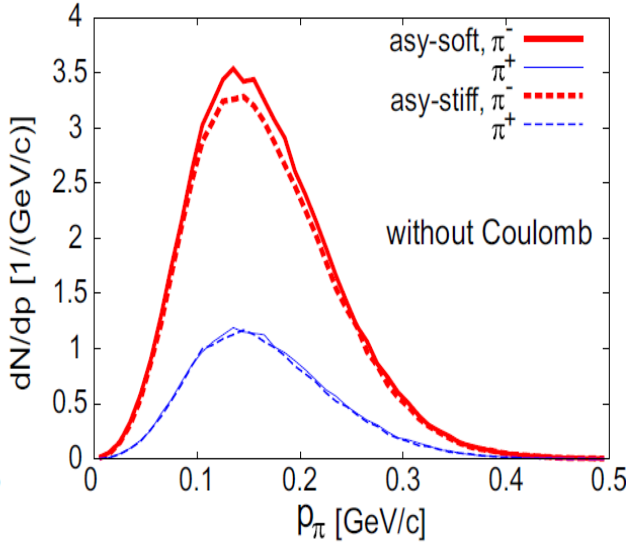
Pion spectra

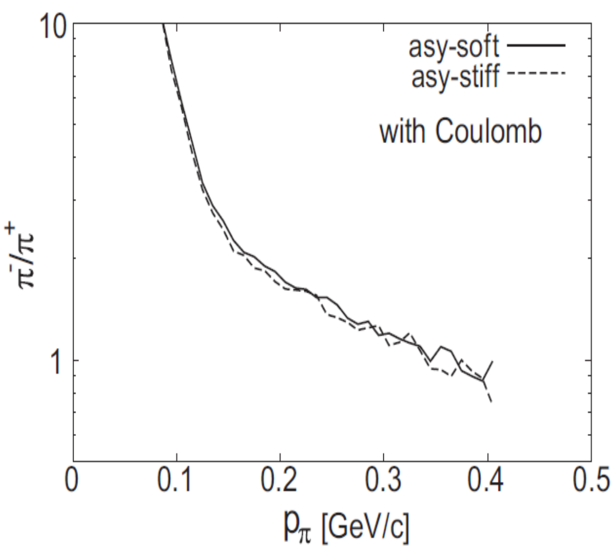
AMD + JAM with cluster (asy-soft)

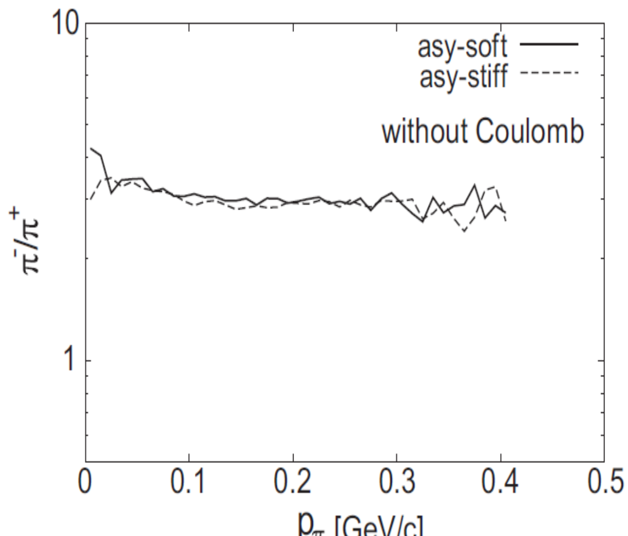
- With Coulomb



- Without Coulomb







- Coulomb effect:
  Acceleration of π^+
  Deceleration of π^-
  → Changes of pion spectra

	π^-	π^+	π^-/π^+
with Coulomb	0.577	0.192	**3.01(1)**
w/o Coulomb	0.582	0.193	**3.02(1)**

→ Coulomb effect has almost no effect on the pion multiplicities and the pion ratio.

Clusters at high density?

In the calculation, cluster correlation played important roles for the pions.
But, in the high density region, should cluster correlations really exist?

3 Options: Treatment of cluster correlations

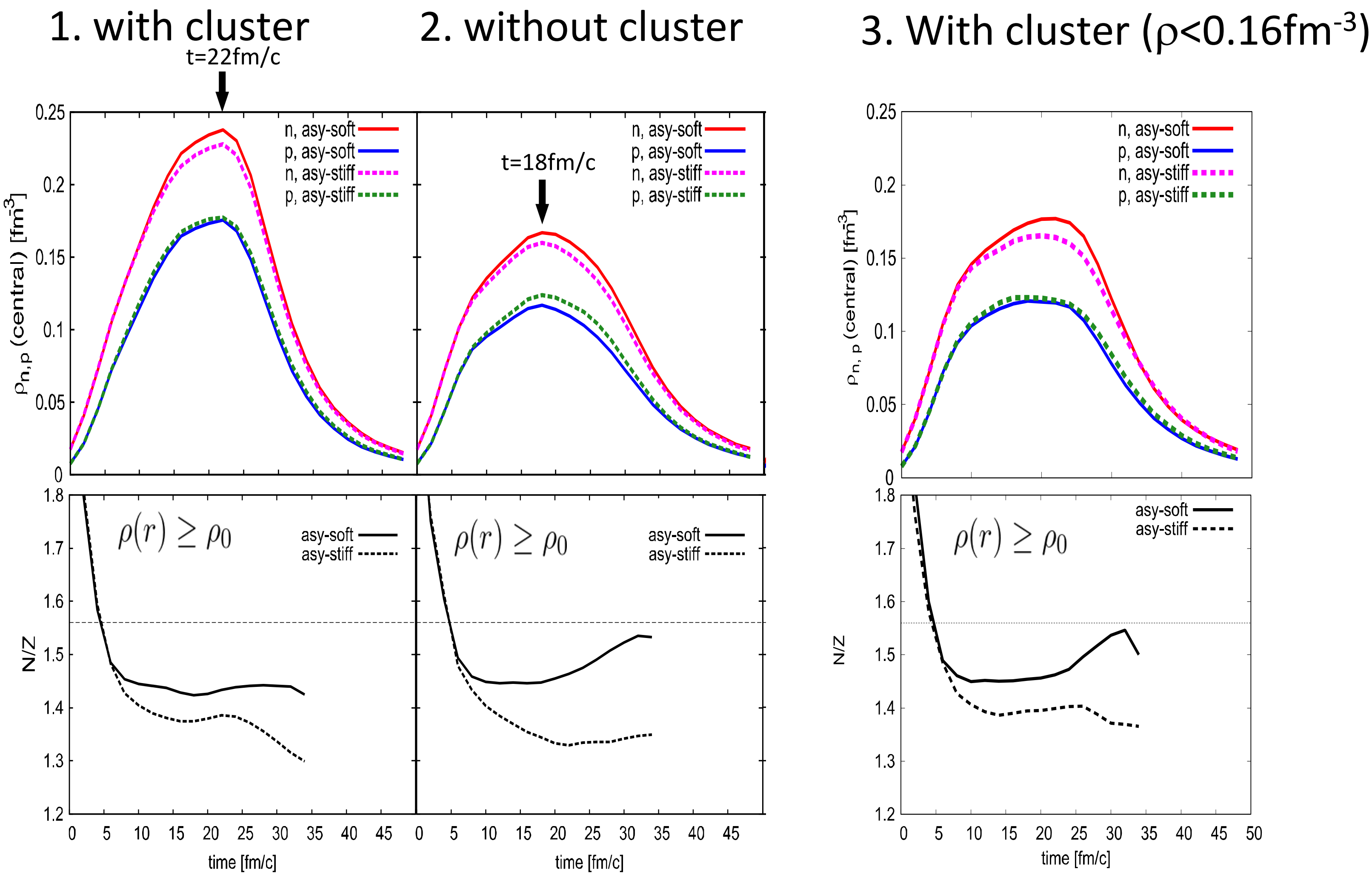
1. With cluster
   Clusters are formed at any density.

2. Without cluster
   Clusters are not formed at all.

NEW 3. With cluster ($\rho < 0.16$ fm^{-3})
   Clusters are formed in the low density region ($\rho < 0.16$ fm^{-3})
   Clusters are not formed in the high density region ($\rho > 0.16$ fm^{-3})

Preliminary result with cluster ($\rho < 0.16\ fm^{-3}$)

- Dynamics of neutrons and protons



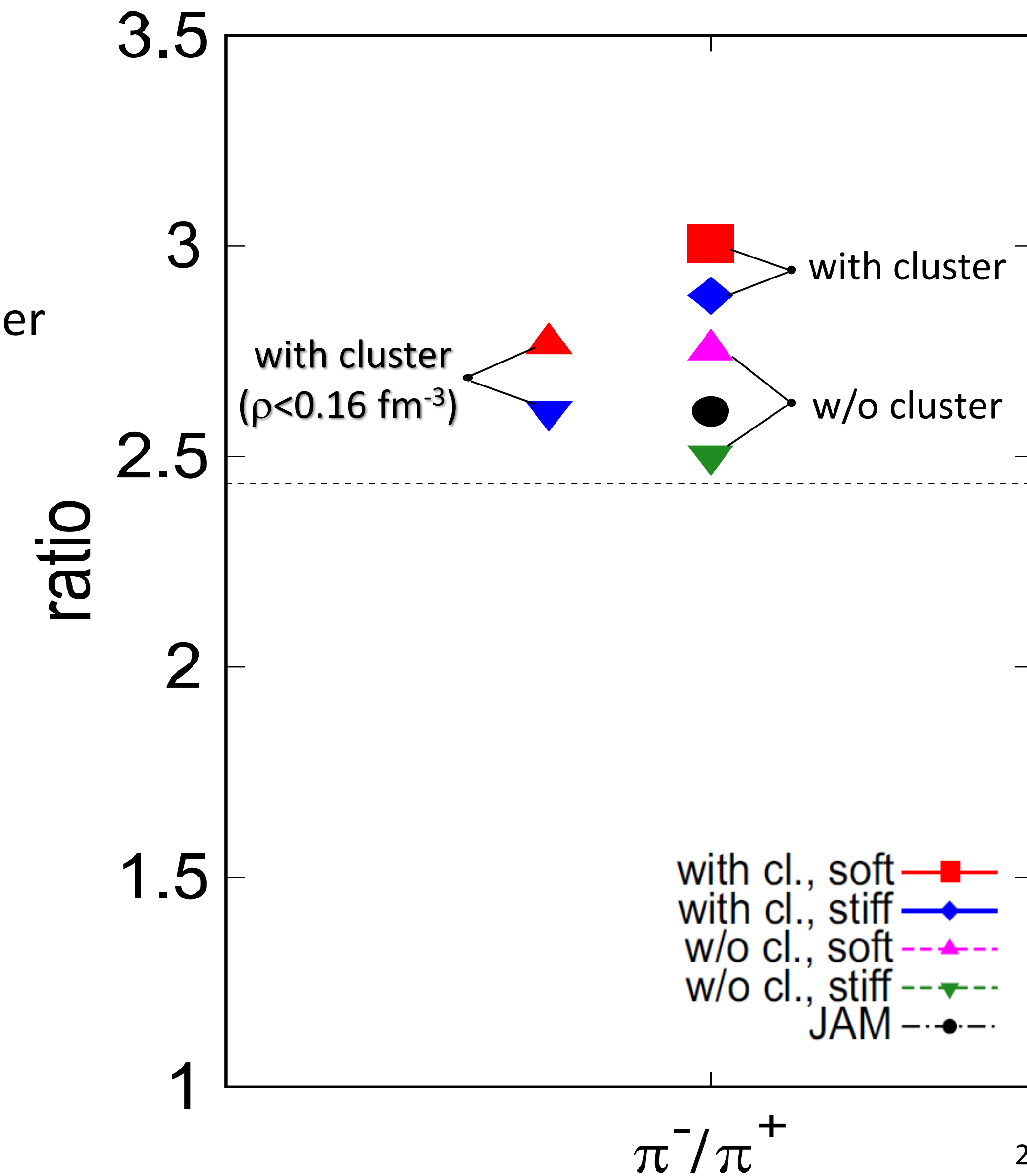
✓ Density maximum is not as high as the case with cluster

Preliminary result with cluster ($\rho < 0.16\ fm^{-3}$)

- Final π^-/π^+ ratio

• With cluster ($\rho < 0.16 fm^{-3}$)

Closer to the case without cluster



Potential for Δ and pion

In JAM, reaction thresholds are the same as in free space.

(The production and absorption reactions for Δ and pions occur in the JAM calculation as in the free space)

Nucleons feel potential in the AMD calculation.

Therefore AMD+JAM assumes

$$\mathrm{NN} \leftrightarrow \mathrm{N}\Delta \qquad\qquad \Delta \leftrightarrow \mathrm{N}\pi$$

$$U_{\tau_1}^{(N)} + U_{\tau_2}^{(N)} = U_{\tau_3}^{(N)} + U_{\tau_4}^{(\Delta)}, \qquad U_{\tau_1}^{(\Delta)} = U_{\tau_3}^{(N)} + U_{\tau_4}^{(\pi)} \qquad \text{for } \tau_1(+\tau_2) = \tau_3 + \tau_4$$

This is equivalent to the choice in the pBUU calculation
c.f. Hong and Danielewicz, PRC 90 (2014) 024605

$$\begin{aligned}
v_{asy}(\Delta^-) &= 2v_{asy}(n) - v_{asy}(p) = 3v_{asy}(n),\\
v_{asy}(\Delta^0) &= v_{asy}(n),\\
v_{asy}(\Delta^+) &= v_{asy}(p) = -v_{asy}(n),\\
v_{asy}(\Delta^{++}) &= 2v_{asy}(p) - v_{asy}(n) = -3v_{asy}(n).
\end{aligned}$$

* Different choice,
cf. Bao-An Li

$$\begin{aligned}
v_{asy}(\Delta^-) &= v_{asy}(n),\\
v_{asy}(\Delta^0) &= \frac{2}{3}v_{asy}(n) + \frac{1}{3}v_{asy}(p) = \frac{1}{3}v_{asy}(n),\\
v_{asy}(\Delta^+) &= \frac{1}{3}v_{asy}(n) + \frac{2}{3}v_{asy}(p) = -\frac{1}{3}v_{asy}(n),\\
v_{asy}(\Delta^{++}) &= v_{asy}(p) = -v_{asy}(n).
\end{aligned}$$